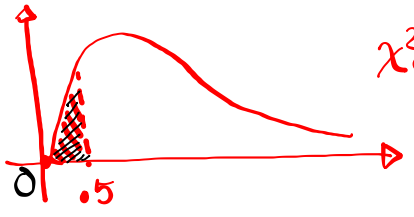
Statistics
Winter 2022
Lecture 16

Extra Credit QZ (Added to exam Score)

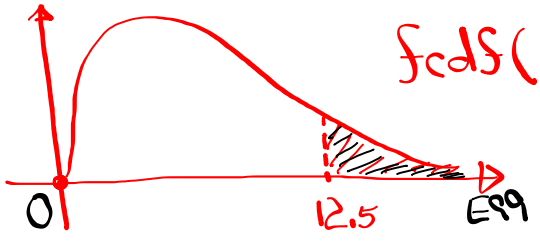
1) Find $P(\chi^2 < .5)$ with $df = 9$.



$\chi^2cdf(0, .5, 9) = 3.04 \times 10^{-5}$ ✓

Lower upper df

2) Find $P(F > 12.5)$ with $Ndf = 5$, $Ddf = 30$.



Lower upper

$Fcdf(12.5, E99, 5, 30)$

$= 1.3 \times 10^{-6}$ ✓

Testing one population standard deviation σ:

$H_0: \sigma = \sigma_0$ vs $H_1: \sigma \neq \sigma_0$ — TTT

$H_0: \sigma \geq \sigma_0$ vs $H_1: \sigma < \sigma_0$ — LTT

$H_0: \sigma \leq \sigma_0$ vs $H_1: \sigma > \sigma_0$ — RTT

we do p-value method only:

CTS $\chi^2 = \dfrac{(n-1)\cdot s^2}{\sigma^2}$, then we use χ^2cdf to find the p-value.

If p-value $> \alpha$ $\Rightarrow$ H_0 valid, H_1 invalid

If p-value $\leq \alpha$ $\Rightarrow$ H_0 invalid, H_1 valid

Final Conclusion:

Reject the claim or FTR the claim.

Given: $H_0: \sigma \leq 10$, claim is H_0, $n = 8$, $s = 12$, $\alpha = .02$

$\Rightarrow df = n - 1 = 7$

Test the claim.

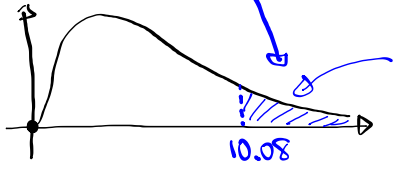
CTS

$H_0: \sigma \leq 10$ claim

$H_1: \sigma > 10$ RTT

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(8-1)\cdot 12^2}{10^2}$$

$$\chi^2 = 10.08$$



Area = P-value

$= \chi^2$cdf$(10.08, E99, 7)$

$= .184$

P-value $>$ α

$.184 > .02$

$\Rightarrow$ H_0 valid & H_1 invalid

Valid claim $\Rightarrow$ FTR the claim

The Math dept claims that Standard deviation of all final exam scores is at least 8. $\sigma \geq 8$

I randomly selected 10 final exams, $n=10$ and standard deviation of their scores was 7.5. $s=7.5$

use this sample to test the claim.

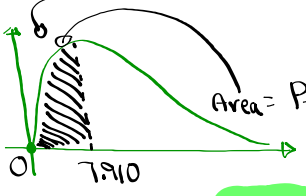
No $\alpha \Rightarrow$ use .05

$H_0: \sigma \geq 8$ claims

$H_1: \sigma < 8$ LTT

CTS $\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(10-1)\cdot 7.5^2}{8^2}$

$\chi^2 = 7.910$



Area = P-value = χ^2cdf(0, 7.910, 9)

= .457

H_0 valid $\rightarrow$ valid claim

H_1 invalid FTR the claim

P-value $>$ α

.457 .05

CNN claims that the standard deviation of ages of all vaccinated people is 12 yrs. $\sigma = 12$

In a sample of 15, $n=15$ vaccinated people, standard deviation of their ages was 8 yrs. $s=8$

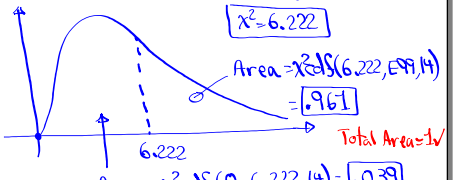
use $\alpha = .1$ to test the claim.

$H_0: \sigma = 12$ claim

$H_1: \sigma \neq 12$ TTT

CTS $\chi^2 = \frac{(n-1)\cdot s^2}{\sigma^2} = \frac{(15-1)\cdot 8^2}{12^2}$

$\chi^2 = 6.222$



Area = χ^2cdf(6.222, E99, 14) = .961

Area = χ^2cdf(0, 6.222, 14) = .039

P-value = 2 * Smaller area

= 2*(.039) = .078

P-value $\leq \alpha \Rightarrow H_0$ invalid, H_1 valid

.078 .1

Invalid claim $\Rightarrow$ Reject the claim.

IS we change α to .07, .06, .05, ..., then

P-value $> \alpha \Rightarrow H_0$ valid $\Rightarrow$ valid claim

$\Rightarrow$ FTR the claim

I randomly selected 8 cars on FWY 210. Here are the speeds:

75	70	80	82
68	65	75	85

Find $\bar{x}$ & S. Round to a whole #

$\bar{x} \approx 75$

$S \approx 7$

$n = 8$

Test the claim that standard deviation of speed of all cars on FWY 210 is 10 mph.

NO α ⇒ use .05

$H_0: \sigma = 10$ claim

$H_1: \sigma \neq 10$ TTT

CTS $\chi^2 = \frac{(n-1)\cdot S^2}{\sigma^2} = \frac{(8-1)\cdot 7^2}{10^2}$

$\chi^2 = 3.43$

P-value

Area $= \chi^2 cdf(3.43, E99, 7)$ = .843

Total = 1 ✓

3.43

Area $= \chi^2 cdf(0, 3.43, 7)$ = .157

P-value = 2 * Smaller area = 2(.157) = .314

P-value > α ⇒ H_0 valid → valid claim

.314 > .05

H_1 invalid

FTR the claim

SG 27 & SG 28 ✓

Comparing Two Population Standard Deviations σ_1 & σ_2:

SG 32

$H_0: \sigma_1 = \sigma_2$	$H_0: \sigma_1 \leq \sigma_2$	$H_0: \sigma_1 \geq \sigma_2$
$H_1: \sigma_1 \neq \sigma_2$	$H_1: \sigma_1 > \sigma_2$	$H_1: \sigma_1 < \sigma_2$
TTT	RTT	LTT

Sample 1	Sample 2
n_1	n_2
S_1	S_2

$S_1 > S_2$

CTS & P-value

STAT TESTS 2-SampFTest

CTS $F = \frac{S_1^2}{S_2^2}$

P-value Fcdf

NdF $= n_1 - 1$

DdF $= n_2 - 1$

Proceed with testing chart, and make final Conclusion about the claim.

Consider the chart below

Sample 1	Sample 2
$n_1 = 8$	$n_2 = 10$
$s_1 = 12$	$s_2 = 8$

1) Verify that $s_1 > s_2$.

$12 > 8$ ✓

2) Use $\alpha = .02$ to test the claim that $\sigma_1 = \sigma_2$.

$H_0: \sigma_1 = \sigma_2$ claim

$H_1: \sigma_1 \neq \sigma_2$ TTT

CTS & P-value

STAT TESTS 2-SampFTest

CTS $F = 2.25$ input: Stats

P-value $P = .256$ ✓

$\sigma_1 \neq \sigma_2$

P-value $>$ α

$.256 > .02$

H_0 valid

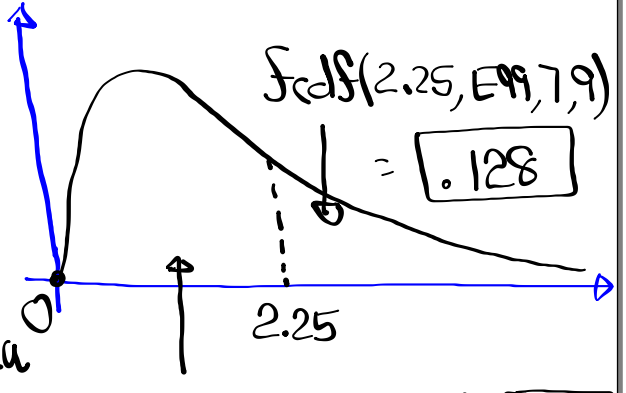
H_1 invalid

Valid claim $\Rightarrow$ FTR the claim.

Sample 1	Sample 2
$n_1 = 8$	$n_2 = 10$
$s_1 = 12$	$s_2 = 8$

TTT

CTS $F = \frac{s_1^2}{s_2^2} = \frac{12^2}{8^2} = 2.25$



P-value = 2 * Smaller area

$= 2(.128)$

$= .256$

fcdf(0, 2.25, 7, 9) = .872

I randomly selected 10 female students, their mean age was 28 yrs with standard deviation 9 yrs.

I also randomly selected 12 male students their mean age was 30 yrs with standard deviation of 6 yrs.

1) Complete the chart below with $S_1 > S_2$

Females	Males
$n_1 = 10$	$n_2 = 12$
$\bar{x}_1 = 28$	$\bar{x}_2 = 30$
$S_1 = 9$	$S_2 = 6$

$S_1 > S_2$

2) Use $\alpha = .02$ to test the claim that $\sigma_1 \neq \sigma_2$.

$H_0: \sigma_1 = \sigma_2$

$H_1: \sigma_1 \neq \sigma_2$ Claim

TTT

CTS $F = 2.25$

P-value $P = .206$ ✓ → P-value $> \alpha$

H_0 valid, H_1 invalid

Invalid claim

Reject the claim

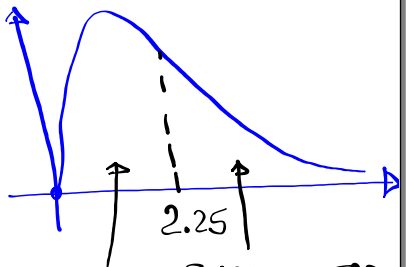
2-SampFTest

inpt: STATS

$\sigma_1 \neq \sigma_2$

Females	Males
$n_1 = 10$	$n_2 = 12$
$\bar{x}_1 = 28$	$\bar{x}_2 = 30$
$S_1 = 9$	$S_2 = 6$
NdF $= 10-1=9$	Ddf $= 12-1=11$

CTS $F = \frac{S_1^2}{S_2^2} = \frac{9^2}{8^2} = 2.25$



Fcdf(0, 2.25, 9, 11)

$= .897$ ✓ Total $= 1$

Fcdf(2.25, E99, 9, 11)

$= .103$ ✓

P-value $= 2 *$ Smaller area

$= 2(.103)$

$= .206$

Morning class Test Scores

90 85 65
100 70 80
95

$n=7, \bar{x}=84, S=13$

Afternoon class Test Scores

100 98 65
60 68 55

$n=6, \bar{x}=74, S=20$

Find $\bar{x}$ & S for each class, Round to a whole #

Afternoon	Morning
$n_1=6$	$n_2=7$
$\bar{x}_1=74$	$\bar{x}_2=84$
$S_1=20$	$S_2=13$

$S_1 > S_2$

Test the claim that $\sigma_1 > \sigma_2$. NO $\alpha \rightarrow .05$

$H_0: \sigma_1 \leq \sigma_2$

$H_1: \sigma_1 > \sigma_2$ claim

RTT

CTS F = 2.367
Pvalue P = .162 ✓

2-SampFTest
inpt: STATS

$\sigma_1 > \sigma_2$

P-Value > α
.162 .05

H_0 valid, H_1 invalid

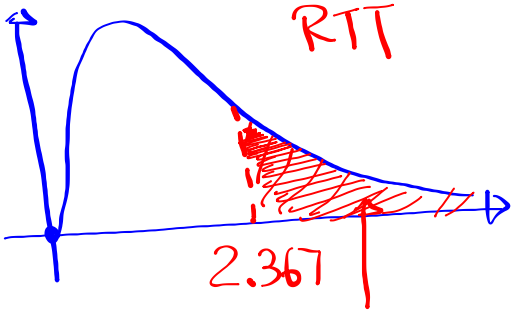
Reject the claim

Afternoon	Morning
$n_1=6$	$n_2=7$
$\bar{x}_1=74$	$\bar{x}_2=84$
$S_1=20$	$S_2=13$

$Ndf = 6-1 = 5$ $Ddf = 7-1 = 6$

CTS

$$F = \frac{S_1^2}{S_2^2} = \frac{20^2}{13^2} = 2.367$$



SE 32 ✓

2-SampFTest(2.367, E99, 5, 6) = P-value = Area

.162

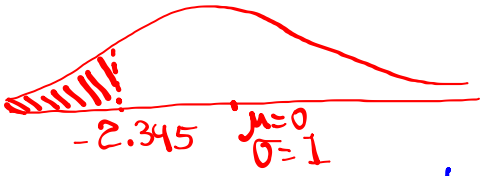
Live QZ 6

1) Given CTS $Z = -2.345$, LTT,

Find P-value. P-value = normalcdf(-E99, -2.345, 0, 1)

= .0095

= .010



2) Given CTS $t = 1.888$, $df = 9$, TTT

Find P-value.

P-value = 2*tcdf(1.888, E99, 9)

= .092